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Physics of Neutronless Fusion Reacting Plasmas

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Abstract

On the basis of present day knowledge of plasma physics and available technologies it is shown that experiments having the purpose to test the thermonuclear burn conditions of plasmas with fusion reactions that do not produce neutrons can be undertaken.

1. Introduction

Recent theoretical developments in plasma physics coupled with novel experimental observations and advances in the technology of high magnetic field plasma devices, have made it possible to think [1, 2] of feasible experiments [3] to study the burning conditions of thermonuclear plasmas such as D–He^3 and D–D, that do not involve tritium as a primary fuel. In fact, in order to acheive this goal, it is necessary to have a plasma confinement configuration that can attain, for instance:

(a) values of $n_0\tau_E$ (n_0 being the peak plasma density and τ_E the energy replacement time) about or higher than 5×10^{14} s cm^{-3};

(b) values of $\langle\beta\rangle = 8\pi\langle p\rangle/\langle B^2\rangle$ ($\langle p\rangle$ being the average plasma pressure and $\langle B^2\rangle/8\pi$ the average magnetic pressure of the confining magnetic field) around 10 percent.

(c) plasma currents around 5 MA or higher, in order to generate the required magnetic fields needed to confine the 14.7 MeV protons produced; and

(d) average plasma temperatures as high as 30 keV.

These objectives can be achieved simultaneously in an axisymmetric toroidal configuration by experiments in which:

– Goal (a) is pursued on the basis of presently known scalings for the plasma thermal conductivity that exhibit a favorable dependence of τ_E on n. Thus it is proposed that peak particle densities exceeding 10^{15} cm^{-3}, can be obtained in a configuration with sufficiently high magnetic fields (e.g., 120 kG) having an adequate area of its transverse cross-section in order to meet the desired $n_0\tau_E$ criterion. Well-confined plasmas with peak density values higher than 10^{15} cm^{-3} have, in fact, been produced already in the Alcator A and C devices at MIT.

– Goal (b) is pursued by adopting a combination of magnetic and geometric parameters, such as the torus aspect ratio, in such a way that during the heating cycle the plasma is maintained within the relatively broad range of plasma parameters where new favorable conditions for macroscopic stability (against both ballooning and internal kink modes) have been found.

– Existing high magnetic field technologies make it possible to produce plasma columns with high currents, without any of the known macroscopic plasma instabilities,

– Goal (d) is acheived by a RF heating system supplementing ohmic heating in order to bring a deuterium–tritium plasma to ignition conditions. In fact, tritium can be used as a "match" [1, 2] to raise the plasma temperature, and, as this temperature increases, is gradually replaced [3] as main fuel by He^3 or D.

The most convenient frequency for the auxiliary heating system appears to be that corresponding to the first harmonic of the cyclotron frequency of He^3, and, at the same time, to the second harmonic of the cyclotron frequency of tritium. We note that the effectiveness of ion cyclotron heating in plasmas with two species of ions has been well demonstrated in several experiments carried out on the most advanced existing toroidal devices.

In this connection we notice that the usually-known ignition conditions (based on the assumptions that: (1) the distributions of all components of the background plasma are Maxwellian, and (2) the slowing-down of the charged fusion reaction products is due to Coulomb scattering only) are relaxed when considering that background ions can be promoted in energy by both Coulomb and nuclear elastic collisions [4, 5]. As a consequence of this

(a) fuel particles promoted in energy have a non-negligible self-interaction probability (tail–tail reactions);

(b) fuel particles promoted in energy, and fusion reaction products can fuse with other fuel particles belonging to the Maxwellian part of their distribution ("fast fusion" or "propagating reaction events");

(c) nuclear scattering collisions increase the fraction of energy transferred by charged fusion products to the background ions.

Another related development is the proposal to enhance and control the reaction rate of the fuel species by relative orientation of their nuclear spin. Thus in the case of a D–He^3 reactor it is possible to gain a factor 1.5 in reactivity by injecting and maintaining [6] D and He^3 with spin polarization, along the direction of the magnetic field. At the same time the rate of D–D reactions that produce neutrons can be reduced to such an extent that, if the combination of physics requirements we have indicated can be realized in practice, a nearly neutronless reactor, may no longer be a very distant dream.

2. Accessibility of finite-β states

In order to identify a class of possible experiments with the characteristics indicated earlier, we have developed a program of investigations [8] on the macroscopic stability of axisymmetric toroidal configurations that can reach finite values of β within a reasonably broad range of heating cycles.

We notice that D–He^3 burning conditions where finite-β stable equilibria are needed are reached only for relatively high temperatures and, in the case where tritium is used as a "match", only after D–T ignition is achieved. Therefore

(i) we shall disregard the effects of resistive modes as for the temperatures we consider they are not important,

(ii) we shall assume that the transition from low-β regimes, where D–T ignition can occur, to finite-β regimes is sufficiently fast that the plasma column goes through a sequence of flux conserving equilibria.

One of the results of the theoretical analysis that has been carried out is that a new region of macroscopic stability, in the relevant parameter space, can be reached under finite-β conditions [8]. To illustrate this we introduce the quantity

$$G \simeq \beta \left(\frac{L}{2\pi}\right)^2 \frac{1}{Rr_p} \qquad (2.1)$$

that was identified in the early theory of the so-called "ballooning" modes described by the ideal MHD approximation [9]. Here $\beta = 8\pi p/B^2$, R is the radius of magnetic $r_p = -1/(\mathrm{d}\ln p/\mathrm{d}r)$, and L is the periodicity distance of the magnetic field intensity following a given line of force (the so-called "connection" length). For an axisymmetric toroidal configuration

$$L = 2\pi q R$$

where $q = 2\pi/\iota$ and ι is the rotational transform. A second relevant parameter that measures magnetic shear is

$$\hat{s} = \frac{\mathrm{d}\ln q}{\mathrm{d}\ln r} \qquad (2.2)$$

considering, for simplicity, configurations with circular cross sections in which the variable r denotes the magnetic surface. Referring to Fig. 1 we notice that a confinement configuration, characterized by given radial profiles of the plasma current density and pressure, is represented by a curve in the ($\hat{s}$, G) plane. Therefore, one of the objectives that has been pursued is to find a sequence of configurations represented by ($\hat{s}$, G) curves that do not encounter any instability region. The results of this type of investigation have been encouraging and are discussed in some detail in [8] and [10]. One of the obtained indications is that in order to preserve stability, the value q_0 of q on the magnetic axis should exceed unity when proceeding through finite values of β. This is not a severe difficulty as during D–T ignition it is not necessary to maximize the plasma current in order to exploit ohmic heating, but it is sufficient to have the current needed to confine the charged fusion reaction products.

The next question that arises is whether, once an equilibrium that is stable and has $q_0 > 1$ is reached without crossing the instability region, this can relax as a result of the effects of finite electrical resistivity towards a configuration with values of q_0 below unity [8], while maintaining the stability against ideal MHD ballooning modes. For this reason an effort has been made to investigate the stability of ideal MHD internal kink modes, with prevalent poloidal wave number $m^0 = 1$. The obtained results indicate that, like for ballooning modes, also for these modes [10] there exist a finite-β stability region. In addition this region can be reached by the same type of heating process, in which q_0 is raised above unity after the initial ohmic heating phase, that avoids the instability of ballooning modes (see Fig. 2).

The available experimental information on energy and particle transport is very limited at this time for finite-β regimes and concurrent investigations are being carried out by the Doublet III device of General Atomic, the PLT and PDX devices of Princeton, the ISX-B device of Oak Ridge and the T-11 device of the Kurchatov Institute. Experiments carried out on the Torus II device of Columbia have achieved $\beta_0 \sim 45\%$ without any manifest macroscopic instability [12], but have not provided information yet on transport.

Another relevant question is whether modes inducing magnetic reconnection and not described by the ideal MHD theory such as those with poloidal wave number $m_0 = 1$, that produce internal disruptions, and those, e.g., with $m_0 = 2$, that lead to major disruptions are likely to be excited in the regimes of interest for this work. The relevant analysis [13, 14] indicates that these modes are stable under realistic distributions of the electron temperature and of the current density. However the validity of the adopted linearized approximation is very limited given the fact that the region where reconnection can take place is of the order of the ion gyroradius [15].

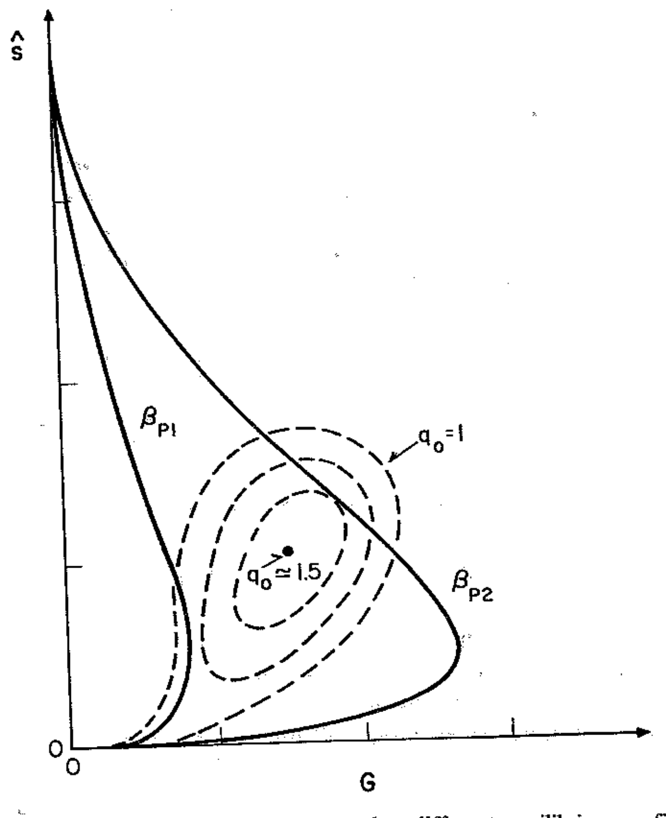

Fig. 1. The curves $\hat{s} = f(G)$ correspond to different equilibrium profiles characterized by different values of β. The variable $\hat{s}$ and G are defined in Section 2. The shaded area correspond to the region in which ideal MHD ballooning modes are unstable. In a configuration with finite aspect ratio, this region vanishes if $q_0 \equiv q(\psi = \psi_0)$, $\psi = \psi_0$ being the magnetic axis, is sufficiently higher than unity.

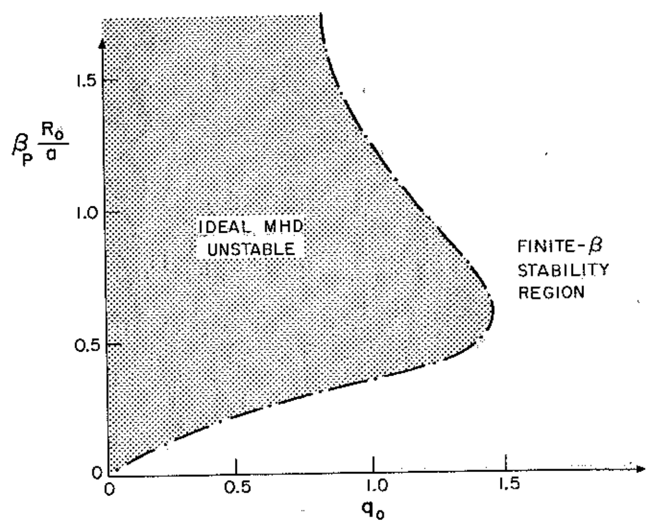

Fig. 2. Permissible region, where neither ideal MHD ballooning modes nor internal kink modes are excited. Here $q_0 = q(\psi = \psi_0)$ and β_p is the so-called poloidal-β. (see Section 2 and [10]).

Finally, we notice that another parameter, that we may consider in order to measure the degree of extrapolation from present day experiments, is the degree of collisionality. Then we may argue that this is not substantially more severe than that required to prove D–T ignition in low particle density [16], large-scale experiments such as those contemplated for the existing fusion program. In fact, the relevant parameter, ν_{**}, that corresponds to the ratio of the torus major radius, R_0, to the particle mean free path is proportional to:

$$\nu_{**} \propto \frac{nR_0}{T^2}$$

Thus, if we take $T_0 \sim 13$ keV, $n_0 \simeq 5 \times 10^{13}\,\mathrm{cm}^{-3}$, and $R_0 \simeq 3$ m, for a low-density D–T igniting experiment, while $T_0 \simeq 65$ keV, $n_0 \sim 2 \times 10^{15}\,\mathrm{cm}^{-3}$, and $R_0 \simeq 1$ m, for the corresponding D–He^3 experiment, we can see that the ratio of the two relevant values of ν_{**} is not far from unity.

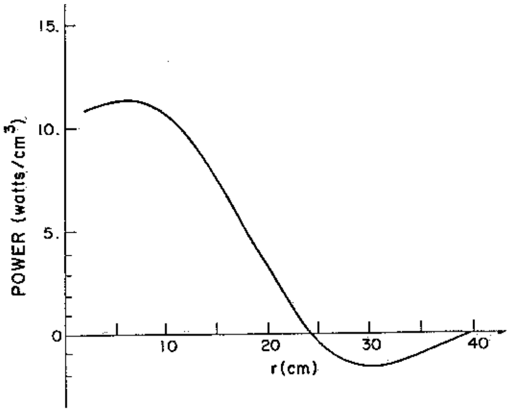

Fig. 3. Example of power loss profile [20] obtained for a toroidal plasma column with parameters close to that of the Candor devices considered in Section 7, by the transport code SNECTR (courtesy of S. Tamor) see Section 4). The reflection coefficient of the plasma chamber is assumed to be 0.9 and the "hole" fraction, of its surface, 0.05.

3. Electron thermal energy transport

In present day experiments on toroidal plasmas most of the plasma thermal energy is lost through the electrons. In the case where the prevalent electron heating is ohmic, the observed electron temperature profiles can in fact be well reproduced numerically by adopting an appropriate set of transport coefficients that are in part collisional and in part anomalous. However, in the case where most of the electron heating is not ohmic and the relevant energy deposition profile is different from that (nearly gaussian) of the plasma current denstiy, the observed effective electron thermal conductivity appears to be consistently higher than in the case of prevalent ohmic heating. An effective diffusion coefficient that can reproduce both the ohmic and the (few) auxiliary heating experiments analyzed so far is given in the appendix and is a development of that reported earlier in [17]. One of the suggestions made is that when the electron energy deposition profile is not considerably different from that of the plasma current density, as in the case of ohmic heating, the scaling of the electron energy confinement time

$$\tau_e \propto \frac{n^{2/3}}{J_\parallel^{1/3}} a^{5/3} \left(\frac{\langle J_\parallel \rangle}{J_0^0}\right)^{5/3} \tag{3.1}$$

is similar to that observed in experiments with prevalent ohmic heating. Here $\langle J_\parallel \rangle$ is the volume averaged current density and J_0^0 the peak value that would be reached if no internal modes were excited.

In assessing the minimal dimensions of feasible [18] D–He^3 burning experiments, such as those described briefly in Section 5, we have in fact referred to the scaling (3.1).

4. Energy transport by synchrotron radiation

The evaluation of the energy transport by synchrotron radiation [19] is a complex problem given the non-local nature of this type of transport and the necessity to consider the effects of a reflecting wall. In particular a major fraction of the energy is carried by frequencies for which the plasma is not optically thick and therefore it cannot be adequately described by a diffusion approximation [20].

We recall that there exists at least one computer code which solves the full transport equations in realistic plasma confinement configurations. We have relied on the results obtained by the use of the SNECTR and CYTRA codes [20] as a way to verify the approximate estimates that we have made for the average, over the plasma cross-section, rate of energy loss [21]. The toroidal configuration analyzed in [20] has a major radius $R_0 = 120$ cm, $a \times b = 35 \times 50\,\mathrm{cm}^2$, $B_0 \simeq 120$ KG and the following temperature and density distributions:

$$T = 65(1 - x^2/a^2)^{1.5}\,\mathrm{KeV} \text{ and } n = 1.5 \times 10^{15} \times (1 - x^2/a^2)\,\mathrm{cm}^{-3}.$$

The wall reflection coefficient is assumed to be about 90% and 5% of the wall to have "holes". The result is illustrated in Fig. 3, and we note the decrease in radiation emission at the center of the plasma column where the magnetic field is depressed by the plasma pressure. Notice that the outer edge of the plasma column synchrotron radiation is reabsorbed and can be considered as a local heating source.

5. Reference devices

Technical feasibility studies have been carried out for two reference devices (Candor I and II) where dimensions have been identified on the basis of different expectations, one more optimistic than the other, for the confinement parameter [18]. Thus in the case of Candor I, we have:

Major radius	$R_0 = 105$ cm
Toroidal field	$B_T = 130$ KG
Elliptical plasma cross-section	$a \times b = 40 \times 55\,\mathrm{cm}^2$
Maxim plasma current	$I_\parallel \simeq 6$ MA

The best plasma parameters that can be envisioned have;

Peak density	$n_0 \simeq 2 \times 10^{15}\,\mathrm{s\,cm}^{-3}$
Confinement parameter	$n_0\tau_E \simeq 10^{15}\,\mathrm{s\,cm}^{-3}$

If this set of plasma parameters were achieved then D–He^3 ignition would become possible and the total power produced by fusion reactions would be about 100 MW. The estimated auxiliary heating power needed in order to reach at first D–T ignition is about 10 MW.

The larger device (Candor II) corresponds to a 70% increase of the plasma cross-section area and to a doubling of the volume. Therefore all the estimated power levels for the former device are doubled, assuming that all the considered radial profiles for the temperatures, the particle densities, etc., remain unchanged. In particular the analyzed set of parameters is

$R_0 = 120\,\text{cm} \quad a \times b = 50 \times 75\,\text{cm}^2$

$B_T \simeq 130\,\text{KG} \quad I_\parallel \simeq 9\,\text{MA}$

Finally we point out that the possibility to conceive of a power producing D–He^3 reactor system has been considered briefly in [18]. In this connection the adoption of supercooled (e.g., around 25 K) aluminum magnets was proposed.

We notice that a study carried out recently by Brown Boveri Co., on the technology of this type of magnets has reached very encouraging conclusions [22].

6. Heating strategy and neutronless mode of operation

One of the most attractive strategy that have been envisioned in order to raise the plasma temperature to the values needed to test D–He^3 burning involves the following steps:

(1) ohmic heating of a D–T plasma with the highest current that is compatible with the stability of the plasma column;

(2) ion cyclotron heating, with He^3 as a minority species, up to ignition conditions for the adopted D–T mixture;

(3) D–T burning with raising peak plasma temperature. During this phase He^3 is injected into the plasma chamber while tritium is depleted;

(4) D–He^3 burning when the peak temperature achieves the maximum values that are compatible with the (finite-β) macroscopic equilibrium and stability conditions of the plasma column. In this phase tritium should be almost completely burned out.

In principle the use of tritium could be avoided and D–He^3 burning conditions could be reached by adopting a suitable ion cyclotron heating system. However, it has been estimated that the cost of the necessary tritium handling facility is smaller than that of the neeeded ion cyclotron heating system.

The energy balance that is realized at the beginning of the D–He^3 burning phase, with a 50–50% mixture, has been roughly estimated to correspond to the following channels of energy loss:

– convection and electron thermal conductivity	53%
– bremstrahlung emission	33%
– synchrotron emission	10%
– neutrons	4%

The synchrotron emission was evaluated assuming 90% reflectivity with 5% of the plasma chamber taken up by access ports ("holes") as indicated in Section 4. The quoted power balance refers to the time where the temperature that can be maintained by D–He^3 burning has just been achieved [3, 7] and the indicated neutron emission is contributed both by D–D reactions and by reactions involving leftover unburned tritium.

In a pure 50–50% mixture of D–He^3 the fraction of power emitted as neutrons has been estimated to amount to about 1% of the total. However this fraction can be reduced by injecting deuterium atoms that are spin polarized in the direction of the magnetic field. If the spin polarization can be maintained through the plasma discharge the deuterium–deuterium reactivity can be made negligible. At the same time, if the spin of the He^3 nuclei are also parallel to that of the deuterous the D–He^3 reaction rate can be enhanced by a factor 3/2, as indicated earlier [6].

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Appendix: Electron thermal energy transport

In this Appendix we discuss the general characteristics of a class of transport coefficients that can be used to describe the electron energy confinement properties of high temperature plasmas. The point of view that is adopted is that these transport coefficients are the results of a set of global conditions for the existence of self-consistent equilibria [1A, 2A] and that:

(1) the current density distribution tends to relax into a profile that has a maximum at the center of the plasma column and is monotonically decreasing toward the column edge;

(2) the class of microscopic processes that allow the current density distribution to relax toward its "optimal" profile is characterized by the dimensionless "number"

$$C_\mu = \omega_{pi} d_e^2 \times \frac{1}{\nu_{ee}\lambda_e^2}$$

where ω_{pi} is the appropriate ion plasma frequency, $d_e = c/\omega_{pe}$ is the electron inertia skin depth, $\lambda_e = V_{the}/\nu_{ee}$ is the electron-electron collision mean free path and $\nu_{ee}\lambda_e^2$ is the diffusion coefficient that represents the longitudinal electron viscosity;

(3) the "optimal" electron energy deposition profile, for which the minimum rate of electron thermal energy diffusion is produced, is close to that of the current density profile, typical of ohmic heating.

We assume that the current density profile, over a significant part of the plasma column, can be described by

$$J_\parallel = J_a \exp\left\{-\frac{q_s}{q_0^0}[I(\xi) - 1]\right\} \tag{1.A}$$

where $\xi \equiv r^2/a^2$, $I(\xi = 1) = 1$, $I(\xi = 0) = 0$, and

$$\frac{\langle J_\parallel \rangle}{J_0^0} = \frac{q_0^0}{q_s} = \int_0^1 \exp\left[-\frac{q_s}{q_0^0} I(\xi)\right] d\xi \tag{2.A}$$

Now we write

$$q_0^0 = 1.6 \frac{B_T}{RJ_0^0}$$

where B_T is the applied toroidal field (in Gauss), R is the torus major radius (in cm) and J_0^0 is measured in A cm^{-2}. Thus, when $q_0^0 > 1$, J_0^0 is the peak current density that can be actually achieved. When $q_0^0 < 1$, J_0^0 is the value that *would* be achieved if *no* internal modes that redistribute the plasma current density were excited. Therefore when $q_0^0 < 1$ and the conditions (e.g., collisionality regimes) for the excitation of $m^0 = 1$, $n^0 = 1$ modes exist, the current density profile is not stationary and is not described by eq. (1.A) in a region $0 < r < r_I < a$ that is affected by these modes. Here m^0 and n^0 indicate the poloidal and toroidal wave numbers, respectively, when referring to a toroidal equilibrium configuration. From now on we shall assume for simplicity that $(r_I/a)^2$ is considerably smaller than unity and that the global electron energy confinement does not depend on the rate of transport over the region $0 \leqslant r \leqslant r_I$.

Since we consider high electron temperature regimes, the collisional electrical resistivity along the magnetic field, can be written as

$$\eta_\parallel = \eta_{cl} F_T(r) \tag{3.A}$$

where $F_T(r) \geqslant 1$ is a function of $(r/R)^{1/2}$, R being the torus major radius, and of $\nu_{e*} = \nu_{eT}/\hat{\omega}_{be}$, ν_{eT} being the effective average collision frequency of trapped electrons and $\hat{\omega}_{be}$ their average bounce frequency. In particular $F_T(r = 0) = F_T(r = a) = 1$. Then, since $E_\parallel \simeq$ const in stationary conditions, the electron temperature profile is related to that of the current density by

$$T_e = T_e(r = a)(F_T)^{2/3} \exp\left\{-\frac{2}{3}\frac{q_s}{q_0^0}\left[I(\xi) - 1\right]\right\} \tag{4.A}$$

and it is broader than $J_\parallel^{2/3}$ in the central region where $F_T(r) > 1$. Here $E_\parallel$ is the current driving electric field.

We also consider regimes for which most of the electrons thermal energy is lost through their thermal conductivity. In the presence of significant auxiliary heating of the electrons that we represent by the specific power $S_e(r)$, we write the relevant thermal energy balance equation as

$$S_e(r) + E_\parallel J_\parallel = -\frac{1}{r}\frac{d}{dr}\left[nD_J r\left(\frac{2}{3}\frac{T_e}{J_\parallel}\frac{dJ_\parallel}{dr}\right)\right] \tag{5.A}$$

Then if we require that the current density profile be of the form (1.A), we obtain

$$D_J = \frac{J_*(r)}{ne}(\tfrac{3}{8}a) \times \left(\frac{eE_\parallel a}{\bar{l}T_e}\frac{q_0^0}{q_s}\right)\left[1 + \frac{S_*(r)}{E_\parallel J_*}\right] \tag{6.A}$$

$$J_*(r) \equiv \frac{2}{r^2}\int_0^r dr\, rJ_\parallel(r) \quad \text{and} \quad S_* \equiv \frac{2}{r^2}\int_0^r S_e(r) r\, dr \tag{7.A}$$

Equation (6.A) ensures that the current density profile is nearly Gaussian and gives the correct functional form of the derived diffusion coefficient. However eq. (6.A) does not provide the specific expression of D_J that is needed for the actual numerical evaluation of the electron temperature.

We also note [3A] that

$$\frac{q_s}{q_0^0} = \ln\left(\frac{\langle J_\parallel \rangle \eta_{cl}(r = a)}{E_\parallel}\frac{q_s}{q_0^0}\right) \tag{8.A}$$

and notice that the argument of the indicated logarithm is a large number. Thus a reasonable error in the estimate of $T_e(r = a)$ does not affect seriously that of q_s/q_0^0.

We observe that in this case we no longer have a diffusion equation for the electron thermal energy and we may refer to the quantity D_{eff}^e defined as

$$D_{eff}^e \simeq D_J \times \left(\frac{d \ln J_\parallel^{2/3}}{d \ln T_e}\right),$$

when comparing the transport model that is being discussed with the reported analyses fo the energy balance in ongoing confinement experiments.

In the case where only ohmic heating is present and $S_e = 0$, we have specified D_J by expressing $eE_\parallel a q_0^0/(\alpha_D T_e q_s)$ in terms of C_μ, that is

$$\frac{eE_\parallel a}{\bar{l}T_e} = \frac{\epsilon_c}{8\pi}\left(\frac{q_s}{6q_0^0}\right) C_\mu^{2/5} \tag{9.A}$$

Thus we can define

$$E_0 \equiv \epsilon_c \left(\frac{q_s}{6q_0^0}\right)\frac{\bar{\bar{V}}}{2\pi R}\left(\frac{R}{4a}\right) \tag{10.A}$$

where

$$\bar{l}T_e C_\mu^{2/5} \equiv e\bar{\bar{V}}$$

and we have

$$l = \int_0^\xi e\bar{\bar{V}}/(T_e C_\mu^{2/5})\, d\xi \tag{11.A}$$

The condition $l(\xi = 1) = 1$ implies that

$$e\bar{\bar{V}} = 1 \Big/ \int_0^1 d\xi/(T_e C_\mu^{2/5}) \tag{12.A}$$

and $E_\parallel \propto (\bar{n}^{1/5}/a)(q_0/q_0^0)$.

Then the expression for D_{th}^{e0}, the thermal diffusion coefficient that is appropriate when only ohmic heating is present, can be written as [4A]

$$D_{th}^{e0} = \epsilon_c \frac{J_*(r)}{ne} \Delta_c \qquad (13.A)$$

where

$$\Delta_c = \tfrac{1}{8} a \times \frac{1}{16\pi} C_\mu^{2/5} \qquad (14.A)$$

$\epsilon_c \simeq 0.25$ is a numerical coefficient that was derived from fitting one set of consistent experimental data with eq. (13), a and Δ_c are measured in cm. We notice that

$$C_\mu \propto \frac{n^{1/2}}{T_e^{5/2}} \left(\frac{Z_i}{A_i}\right)^{1/2}$$

where A_i is the ion mass number, Z_i the charge number.

In the case where $\langle S_e \rangle \neq 0$, we introduce the variables

$$\hat{\mathscr{E}} = \frac{E_0}{\bar{E}_\parallel} \quad \text{and} \quad \hat{S} = \frac{\bar{\bar{S}}_e}{E_0 \bar{J}} \qquad (15.A)$$

and notice that $\hat{\mathscr{E}} \propto T_e^{3/2}(q_s/q_0^0)/(aJ_\parallel)n^{1/5}$, as $\bar{E}_\parallel \equiv \int_0^1 d(r/r_s)E_\parallel$, $\bar{\bar{S}}_e \equiv S_*(r=r_S)$, $\bar{J} = J_*(r=r_S)$ and r_S, defined by $T_e(r=r_S) = T_e(r=0)/5$, represents the edge of the hot region.

Thus we can express the scaling for the electron temperature by relating $\hat{\mathscr{E}}$ to $\hat{S}$. In particular when $\hat{S}\,\hat{\mathscr{E}}$ is large [5A, 6A] we may write

$$\hat{\mathscr{E}}^{2/3} \propto \hat{S}^{1/\Gamma} \qquad (16.A)$$

We notice that $\Gamma \gg 1$ would mean that the electron energy confinement time deteriorates so seriously that the electron temperature does not increase as $\hat{S}$ increase. When $\Gamma = 1$,

$$T_e \propto \frac{S_e}{J_\parallel^{1/3}} \left(a\frac{q_0^0}{q_s}\right)^{5/3} \qquad (17.A)$$

and the electron energy confinement time has the same scaling as in the case where ohmic heating only is present [7A]. Then we may write

$$\hat{\mathscr{E}} = \left[1 + \frac{1}{\gamma}(\hat{S}\hat{\mathscr{E}})\right]^{\frac{3}{5}\Gamma} \qquad (18.A)$$

and, in this case, the expression for D_J is

$$D_J = \epsilon_c \frac{\Delta_c}{ne} \left\{ J_*(r) + \bar{J} \frac{S_*(r)}{\bar{\bar{S}}_e} \gamma(\hat{\mathscr{E}}^{\frac{5}{3}\Gamma} - 1) \right\} \frac{1}{\hat{\mathscr{E}}} \qquad (19.A)$$

In particular we may relate γ to the departure of the energy deposition profile from that typical of ohmic heating. Then, a possible option is to take

$$\gamma - 1 = \left| \int_0^1 d\left(\frac{r}{r_S}\right)^2 \frac{S_e(r)}{\bar{\bar{S}}_e} \frac{\bar{J}}{J_\parallel(r)} - 1 \right|^{1/2} \qquad (20.A)$$

Unfortunately the number of experiments in which the energy balance has been analyzed in detail and ohmic heating is not prevalent is quite limited [8A]. Therefore, although the implications of conjectures (18.A) and (20.A) appear to be verified for $\Gamma \approx 1$, a reasonable precise comparison with a consistent set of experimental data remains to be carried out.

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